Matrices and Determinants

Question1

If $\begin{vmatrix} 1 & 2 & 3-\lambda \\ 0 & -1-\lambda & 2 \\ 1-\lambda & 1 & 3 \end{vmatrix} = A\lambda^3 + B\lambda^2 + C\lambda + D$, **then** $D + A =$

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Options:

A.

1

B.

-4

C.

-5

D.

3

Answer: D

Solution:

We have, $\begin{vmatrix} 1 & 2 & 3-\lambda \\ 0 & -1-\lambda & 2 \\ 1-\lambda & 1 & 3 \end{vmatrix} = A\lambda^3 + B\lambda^2 + C\lambda + D$

Now, $\begin{vmatrix} 1 & 2 & 3-\lambda \\ 0 & -1-\lambda & 2 \\ 1-\lambda & 1 & 3 \end{vmatrix}$

$= 1(-3 - 3\lambda - 2) + (1 - \lambda)(4 + (1 + \lambda)(3 - \lambda))$
$= (-3\lambda - 5) + (1 - \lambda)\left(4 + 3 - \lambda + 3\lambda - \lambda^2\right)$
$= (-3\lambda - 5) + (1 - \lambda)\left(7 + 2\lambda - \lambda^2\right)$
$= (-3\lambda - 5) + \left(7 + 2\lambda - \lambda^2 - 7\lambda - 2\lambda^2 + \lambda^3\right)$
$= \lambda^3 - 3\lambda^2 - 8\lambda + 2$

$\therefore \lambda^3 - 3\lambda^2 - 8\lambda + 2 = A\lambda^3 + B\lambda^2 + C\lambda + D$
On comparing both sides, we get
$$A = 1, B = -3, C = -8, D = 2$$
$\therefore \quad D + A = 2 + 1 = 3$

Question2

If $A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ **and** $2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$**, then** $\operatorname{tr}(A) - \operatorname{tr}(B) =$

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Options:

A.

1

B.

2

C.

3

D.

4

Answer: B

Solution:

We have

$$A+2B=\begin{bmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{bmatrix} \qquad \ldots(i)$$

And $\quad 2A-B=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix} \qquad \ldots(ii)$

Now, $2A+4B=\begin{bmatrix}2 & 4 & 0\\ 12 & -6 & 6\\ -10 & 6 & 2\end{bmatrix} \qquad \ldots(iii)$

Substract Eqs. (ii) from (iii), we get

$$5B=\begin{bmatrix}2 & 4 & 0\\ 12 & -6 & 6\\ -10 & 6 & 2\end{bmatrix}-\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}=\begin{bmatrix}0 & 5 & -5\\ 10 & -5 & 0\\ -10 & 5 & 0\end{bmatrix}$$

$$\Rightarrow B=\begin{bmatrix}0 & 1 & -1\\ 2 & -1 & 0\\ -2 & 1 & 0\end{bmatrix}$$

$\therefore$ From Eqs. (ii), we get

$$2A=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}+\begin{bmatrix}0 & 1 & -1\\ 2 & -1 & 0\\ -2 & 1 & 0\end{bmatrix}$$

$$\Rightarrow 2A=\begin{bmatrix}2 & 0 & 4\\ 4 & -2 & 6\\ -2 & 2 & 2\end{bmatrix}\Rightarrow A=\begin{bmatrix}1 & 0 & 2\\ 2 & -1 & 3\\ -1 & 1 & 1\end{bmatrix}$$

$\therefore \operatorname{tr}(A)-\operatorname{tr}(B)$

$=(1-1+1)-(0-1+0)=1+1=2$

Question3

A, C are 3×3 matrices B, D are 3×1 matrices. If $AX=B$ has unique solution and $CX=D$ has infinite number of solutions, then

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Options:

A.

rank of $[A : D]$ = rank of $[C : B]$

B.

rank of A = rank of C

C.

rank of $[A : B]$ < rank of $[B : D]$

D.

rank of $[A : D] \geq$ rank of $[C : B]$

Answer: D

Solution:

We have,

A, C are 3×3 matrices

$B \times D$ are 3×1 matrices

$AX = B$ has unique solution

$CX = D$ has infinite solution.

Since, $AX = B$ have a unique solution.

$\Rightarrow \text{rank}(A) = \text{number of unknowns} = 3$

$\therefore \quad \text{rank}([A : B]) = \text{rank}(A) = 3 \qquad \ldots(i)$

Since, $CX = D$ have infinite many solutions

$\therefore \quad \text{Rank}(C) = \text{not unique} \Rightarrow \text{rank}(C) < 3$

$\Rightarrow \quad \text{Rank}([C : D]) < 3 \qquad \ldots(ii)$

From Eqs. (i) and (ii), we have

$\text{Rank}([A : D]) \geq \text{rank}[(C : B)]$

Question4

A and B are two non-square matrices. If $P = A + B, Q = A^T B, R = AB^T$, then the matrices whose order is equal to the order of A are

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Options:

A.

PQ and QR

B.

RQ and QP

C.

PQ and RP

D.

PQR and RPQ

Answer: C

Solution:

We have, A and B are two non-square matrices such that

$P = A + B, Q = A^T B, R = AB^T$

Since, $A + B$ is defined,

So order of A and B are same.

Let order of A and B be $m \times n$

$\therefore$ Order of A^T is $n \times m$ and order of B^T is $n \times m$

Now, order of $P =$ order of $A + B = m \times n$

Order of $Q =$ order of $A^T B = n \times n$

And order of $R =$ order of $AB^T = m \times m$

Now, order of $PQ = m \times n$

Order of $QR =$ not defined

Order of $RQ =$ not defined

Order of $QP =$ not defined

Order of $RP = m \times n$

$\therefore$ Order of $A =$ order of PQ and RP

Question5

If the augmented matrix corresponding to the system of equations $x + y - z = 1, 2x + 4y - z = 0$ and $3x + 4y + 5z = 18$ is transformed to $\begin{bmatrix} 1 & a & 0 & -1 \\ 0 & 2 & 1 & b \\ 0 & 0 & c & 32 \end{bmatrix}$ then $\sqrt{a+b+c} =$

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Options:

A.

1

B.

4

C.

9

D.

16

Answer: B

Solution:

Given equations are $x + y - z = 1$,

$2x + 4y - z = 0$ and $3x + 4y + 5z = 18$

The augmented matrix for the system is

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 2 & 4 & -1 & 0 \\ 3 & 4 & 5 & 18 \end{bmatrix}$$

We need to transform this matrix into

$$B = \begin{bmatrix} 1 & a & 0 & -1 \\ 0 & 2 & 1 & b \\ 0 & 0 & c & 32 \end{bmatrix}$$

Now, perform the row operation for matrix A, we get

$R_2 \rightarrow R_2 - 2R_1$ and $R_3 \rightarrow R_3 - 3R_1$

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 2 & 1 & -2 \\ 0 & 1 & 8 & 15 \end{bmatrix}$$

Now, $R_3 \rightarrow R_3 - \frac{1}{2}R_2$

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 2 & 1 & -2 \\ 0 & 0 & 7.5 & 16 \end{bmatrix}$$

Apply $R_3 \rightarrow 2R_3$, we get

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & 2 & 1 & -2 \\ 0 & 0 & 15 & 32 \end{bmatrix}$$

Again apply $R_1 \rightarrow R_1 + R_2$, we get

$$A = \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 2 & 1 & -2 \\ 0 & 0 & 15 & 32 \end{bmatrix}$$

Thus, $a = 3, b = -2$ and $c = 15$

$\Rightarrow \quad c = 15$

$\therefore \quad \sqrt{a+b+c} = \sqrt{3+(-2)+15}$

$= \sqrt{1+15} = \sqrt{16} = 4$

Question6

If $\begin{vmatrix} 9 & 25 & 16 \\ 16 & 36 & 25 \\ 25 & 49 & 36 \end{vmatrix} = K$**, then** $K, K+1$ **are the roots of the equation**

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Options:

A.

$x^2 - 13x + 42 = 0$

B.

$x^2 - 15x + 56 = 0$

C.

$x^2 - 19x + 90 = 0$

D.

$x^2 - 17x + 72 = 0$

Answer: D

Solution:

Given, $K = \begin{vmatrix} 9 & 25 & 16 \\ 16 & 36 & 25 \\ 25 & 49 & 36 \end{vmatrix}$

$$\begin{aligned} &= 9\begin{vmatrix} 36 & 25 \\ 49 & 36 \end{vmatrix} - 25\begin{vmatrix} 16 & 25 \\ 25 & 36 \end{vmatrix} + 16\begin{vmatrix} 16 & 36 \\ 25 & 49 \end{vmatrix} \\ &= 9(1296 - 1225) - 25(576 - 625) + 16(784 - 900) \\ &= 9(71) - 25(-49) + 16(-116) \\ &= 639 + 1225 - 1856 = 8 \\ &\therefore K = 8 \end{aligned}$$

So, the roots are K and $K + 1$, i.e., 8 and 9

$$\begin{aligned} \text{Sum of roots } &= 8 + 9 = 17 \\ &= \frac{-b}{a} = \alpha + \beta \end{aligned}$$

And product of roots $= 8 \times 9 = 72$

$= \frac{c}{a} = \alpha\beta$

So, the general form of a quadratic equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$\Rightarrow \quad x^2 - 17x + 72 = 0$

Question7

$$A = \begin{bmatrix} 1 & -3 & -5 \\ -2 & 4 & -6 \\ 7 & -11 & 13 \end{bmatrix}, \text{ then } \sqrt{|\text{adj } A|} =$$

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Options:

A.

64

B.

16

C.

36

D.

216

Answer: A

Solution:

Given, $A = \begin{bmatrix} 1 & -3 & -5 \\ -2 & 4 & -6 \\ 7 & -11 & 13 \end{bmatrix}$

$$|A| = \begin{vmatrix} 1 & -3 & -5 \\ -2 & 4 & -6 \\ 7 & -11 & 13 \end{vmatrix}$$

$$= 1(4 \times 13 - (-6) \times (-11)) - (-3)(-2 \times 13 - (-6) \times 7) - 5(-2 \times (-11) - 7 \times 4)$$
$$= 1(52 - 66) + 3(-26 + 42) - 5(22 - 28)$$
$$= -14 + 48 + 30 = 64$$

Since, $|\text{adj } A| = |A|^{n-1}$, for $n \times n$ matrix A

So, for 3×3 matrix,

$$|\text{adj } A| = |A|^{3-1} = |A|^2$$
$$\therefore \quad \therefore \quad |\text{adj } A| = |A|^2 = (64)^2 = 4096$$
$$\sqrt{|\text{adj } A|} = \sqrt{4096} = 64$$

Question8

If $\Delta_r = \begin{vmatrix} \frac{1}{3r-2} & \frac{2}{3r-5} \\ 0 & \frac{3}{3r+1} \end{vmatrix}$ then $\sum_{r=1}^{33} \Delta_r =$

Options:

A.

0.99

B.

0.33

C.

0.66

D.

0.55

Answer: A

Solution:

$$\begin{aligned}
\Delta r &= \begin{vmatrix} \frac{1}{3r-2} & \frac{2}{3r-5} \\ 0 & \frac{3}{3r+1} \end{vmatrix} \\
&= \frac{1}{(3r-2)} \times \frac{3}{(3r+1)} - \frac{2}{3r-5} \times 0 \\
&= \frac{3}{(3r-2)(3r+1)}
\end{aligned}$$

Using partial fractions

$$\begin{aligned}
& \Delta r = \frac{A}{3r-2} + \frac{B}{3r+1} \\
\Rightarrow \quad & 3 = A(3r+1) + B(3r-2)
\end{aligned}$$

If $\quad r = \frac{2}{3}$, then

$$\begin{aligned}
& \Rightarrow \quad 3 = A\left(3 \times \frac{2}{3} + 1\right) + B\left(3 \times \frac{2}{3} - 2\right) \\
& \Rightarrow 3A = 3 \\
& \Rightarrow \quad A = 1
\end{aligned}$$

If $\quad r = -\frac{1}{3}$, then

$$3 = B\left(3 \times \left(-\frac{1}{3}\right) - 2\right) = B(-1-2)$$
$$\Rightarrow \quad B = -1$$
So, $\Delta r = \frac{1}{3r-2} - \frac{1}{3r+1}$
$$\therefore \sum_{r=1}^{33} \Delta r = \sum_{r=1}^{33}\left(\frac{1}{3r-2} - \frac{1}{3r+1}\right)$$
$$\Rightarrow \left(\frac{1}{1} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{10}\right) +$$
$$\ldots + \left(\frac{1}{97} - \frac{1}{100}\right)$$
$$\Rightarrow 1 - \frac{1}{100} = \frac{99}{100} = 0.99$$

Question9

$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ **and** $B = \begin{bmatrix} x & y \\ 1 & 2 \end{bmatrix}$ **are two matrices such that** $(A+B)(A-B) = A^2 - B^2$ **If** $C = \begin{bmatrix} x & 2 \\ 1 & y \end{bmatrix}$**, then trace** $(C) =$

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Options:

A. 3

B. 5

C. 7

D. 9

Answer: A

Solution:

Given the matrices A and B:

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} x & y \\ 1 & 2 \end{bmatrix}$$

We have the condition:

$(A+B)(A-B)=A^2-B^2$

This implies:

$AB-BA=0 \quad \Rightarrow \quad AB=BA$

Let's compute AB and BA:

$$AB=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}\begin{bmatrix}x & y\\ 1 & 2\end{bmatrix}=\begin{bmatrix}x+2 & y+4\\ 2x+1 & 2y+2\end{bmatrix}$$

$$BA=\begin{bmatrix}x & y\\ 1 & 2\end{bmatrix}\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}=\begin{bmatrix}x+2y & 2x+y\\ 1+4 & 2+2\end{bmatrix}=\begin{bmatrix}x+2y & 2x+y\\ 5 & 4\end{bmatrix}$$

Equating AB and BA, we have:

$$\begin{bmatrix}x+2 & y+4\\ 2x+1 & 2y+2\end{bmatrix}=\begin{bmatrix}x+2y & 2x+y\\ 5 & 4\end{bmatrix}$$

From this, we derive:

$$2x+1=5$$
$$2y+2=4$$

Thus:

$$2x=4 \quad \Rightarrow \quad x=2$$
$$2y=2 \quad \Rightarrow \quad y=1$$

Given matrix C:

$$C=\begin{bmatrix}x & 2\\ 1 & y\end{bmatrix}=\begin{bmatrix}2 & 2\\ 1 & 1\end{bmatrix}$$

The trace of matrix C, denoted as $\operatorname{tr}(C)$, is:

$\operatorname{tr}(C)=x+y=2+1=3$

Question10

If $x=k$ satisfies the equation $\begin{vmatrix}x-2 & 3x-3 & 5x-5\\ x-4 & 3x-9 & 5x-25\\ x-8 & 3x-27 & 5x-125\end{vmatrix}=0$, then $x=k$ also satisfies the equation

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Options:

A. $x^2 + x - 2 = 0$

B. $x^2 - x - 6 = 0$

C. $x^2 - 2x - 8 = 0$

D. $x^2 + 2x - 3 = 0$

Answer: D

Solution:

To solve the given determinant equation, we start by simplifying the expression:

$$\begin{vmatrix} x-2 & 3(x-1) & 5(x-1) \\ x-4 & 3(x-3) & 5(x-5) \\ x-8 & 3(x-9) & 5(x-25) \end{vmatrix} = 0$$

This simplifies to:

$$\begin{vmatrix} x-2 & x-1 & x-1 \\ x-4 & x-3 & x-5 \\ x-8 & x-9 & x-25 \end{vmatrix} = 0$$

Next, perform column operations to simplify the determinant. Subtract the second column from the first:

$$C_1 \to C_1 - C_2 \implies \begin{vmatrix} -1 & x-1 & x-1 \\ -1 & x-3 & x-5 \\ 1 & x-9 & x-25 \end{vmatrix} = 0$$

Now, apply row operations:

Add the third row to the first row: $R_1 \to R_1 + R_3$

Add the third row to the second row: $R_2 \to R_2 + R_3$

This results in:

$$\begin{vmatrix} 0 & 2x-10 & 2x-26 \\ 0 & 2x-12 & 2x-30 \\ 1 & x-9 & x-25 \end{vmatrix} = 0$$

To solve, expand the determinant by the first column:

$(2x - 30)(2x - 10) - (2x - 12)(2x - 26) = 0$

This simplifies to:

$2(x - 15) \cdot 2(x - 5) - 2(x - 6) \cdot 2(x - 13) = 0$

Further simplifying, we have:

$(x - 15)(x - 5) - (x - 6)(x - 13) = 0$

Solving this, we calculate:

$x^2 - 20x + 75 - (x^2 - 19x + 78) = 0$

This leads to:

$-x - 3 = 0 \implies x = -3$

Therefore, $x = -3$ satisfies the equation $x^2 + 2x - 3 = 0$.

Question11

If A is a non-singular matrix, then $\operatorname{adj}\left(A^{-1}\right) =$

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Options:

A. $(\operatorname{adj} A)^{-1}$

B. $\frac{1}{|A|} A^{-1}$

C. $|A| A^{-1}$

D. $|A| A$

Answer: A

Solution:

We start with the well-known formula for any invertible matrix A:

$A \cdot \operatorname{adj}(A) = |A| I.$

Since A is non-singular ($|A| \neq 0$), we can write

$\operatorname{adj}(A) = |A|\, A^{-1}.$

Now, replace A with A^{-1}. Then, applying the same formula for A^{-1}, we have:

$A^{-1} \cdot \operatorname{adj}(A^{-1}) = |A^{-1}|\, I.$

Recall that $|A^{-1}| = \frac{1}{|A|}$. Thus,

$A^{-1} \cdot \operatorname{adj}(A^{-1}) = \frac{1}{|A|}\, I.$

Multiplying both sides on the left by A (which is valid since A is invertible) gives:

$A \cdot A^{-1} \cdot \operatorname{adj}(A^{-1}) = A \cdot \frac{1}{|A|}\, I.$

Since $A \cdot A^{-1} = I$, this simplifies to:

$\text{adj}(A^{-1}) = \frac{1}{|A|} A.$

On the other hand, we earlier had:

$\text{adj}(A) = |A|\, A^{-1}.$

Taking the inverse of both sides (and recalling that for any nonzero scalar c and invertible matrix B, $(cB)^{-1} = \frac{1}{c} B^{-1}$), we get:

$(\text{adj}(A))^{-1} = (|A|\, A^{-1})^{-1} = \frac{1}{|A|} (A^{-1})^{-1} = \frac{1}{|A|} A.$

Thus, we obtain:

$\text{adj}(A^{-1}) = (\text{adj}(A))^{-1}.$

Therefore, the correct option is:

Option A – $(\text{adj } A)^{-1}$**.**

Question12

If the homogeneous system of linear equations $x - 2y + 3z = 0, 2x + 4y - 5z = 0, 3x + \lambda y + \mu z = 0$ has non-trivial solution, then $8\mu + 11\lambda =$

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Options:

A. 2

B. 6

C. -6

D. -2

Answer: B

Solution:

$$\begin{vmatrix} 1 & -2 & 3 \\ 2 & 4 & -5 \\ 3 & \lambda & \mu \end{vmatrix} = 0$$

$1(4\mu + 5\lambda) + 2(2\mu + 15) + 3(2\lambda - 12) = 0$

$\Rightarrow 4\mu + 5\lambda + 4\mu + 30 + 6\lambda - 36 = 0$

$11\lambda + 8\mu - 6 = 0$

$\therefore \quad 8\mu + 11\lambda = 6$

Question13

If $\frac{x^2}{2x^4+7x^2+6} = \frac{Ax+B}{x^2+a} + \frac{Cx+D}{ax^2+3}$, then $A + B + C - 2D =$

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Options:

A. $2a$

B. $-2a$

C. $-4a$

D. $4a$

Answer: D

Solution:

$\frac{x^2}{2x^4+7x^2+6} = \frac{x^2}{(x^2+2)(2x^2+3)}$

$= \frac{Ax+B}{x^2+a} + \frac{Cx+D}{ax^2+3}$

which gives, $a = 2$

$$\frac{x^2}{(x^2+2)(2x^2+3)} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{2x^2+3}$$

$x^2 = (Ax+B)\left(2x^2+3\right) + (Cx+D)\left(x^2+2\right)$

$0 = 2A + C$

$1 = 2B + D$

On solving these equations, we get .

$$0 = 3A + 2C$$

$$0 = 3B + 2D$$

$$A = C = 0$$

$$B = 2, D = -3$$

$$\therefore \quad A+B+C-2D = 0+2+0-2(-3)$$

$$= 2 + 6 = 8$$

$$= 4(2) = 4a$$

Question14

$A = [a_{ij}]$ is a 3×3 matrix with positive integers as its elements. Elements of A are such that the sum of all elements of each row is equal to 6 and $a_{22} = 2$.

If $\mathrm{a}_{ij} = \begin{cases} \mathrm{a}_{ij} + \mathrm{a}_{ji}, & j = i + 1 \text{ when } i < 3 \\ \mathrm{a}_{ij} + \mathrm{a}_{ji}, & j = 4 - i \text{ when } i = 3 \end{cases}$ **for** $i = 1, 2, 3$, **then** $|\mathrm{A}| =$

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Options:

A. 6

B. 18

C. 3

D. 12

Answer: D

Solution:

Let's consider the 3×3 matrix $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ where each element is a positive integer and each row sums to 6, with $a_{22} = 2$.

The matrix properties provide the following conditions:

$a_{22} = a_{23} + a_{32}$ implies $a_{23} + a_{32} = 2$.

$a_{11} = a_{12} + a_{21}$.

$a_{33} = a_{31} + a_{13}$.

Substituting the conditions about row sums, we get:

$a_{11} + a_{12} + a_{13} = 6$.

$a_{21} + a_{22} + a_{23} = 6$.

$a_{31} + a_{32} + a_{33} = 6$.

From $a_{23} + a_{32} = 2$, if $a_{23} = 1$, then $a_{32} = 1$.

From the second row sum, we have: $a_{21} + 2 + 1 = 6$ which gives $a_{21} = 3$.

Given $a_{11} = a_{12} + a_{21}$ and the first row sum,

with $2a_{12} + a_{13} = 3$,

choosing $a_{12} = 1$ and $a_{13} = 1$, results in $a_{11} = 4$.

Finally, using the third row equation $2a_{31} + a_{32} + a_{13} = 6$,

substituting $a_{32} = 1$ and $a_{13} = 1$ results in $a_{31} = 2$.

Now, from $a_{31} + a_{32} + a_{33} = 6$, with $a_{31} = 2$ and $a_{32} = 1$,

results in $a_{33} = 3$.

Matrix A is:

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

To find the determinant $|A|$, we calculate:

$|A| = 4(2 \cdot 3 - 1 \cdot 1) - 1(3 \cdot 3 - 1 \cdot 2) + 1(3 \cdot 1 - 2 \cdot 2)$

$= 4(6 - 1) - 1(9 - 2) + 1(3 - 4)$

$= 4 \cdot 5 - 1 \cdot 7 + 1 \cdot (-1)$

$= 20 - 7 - 1 = 12$

Thus, the determinant $|A| = 12$.

Question15

If $|\text{adj } A| = x$ and $|\text{adj } B| = y$, then $\left|(\text{adj}(AB))^{-1}\right| =$

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Options:

A. $\frac{1}{x}+\frac{1}{y}$

B. xy

C. $\frac{1}{xy}$

D. $x+y$

Answer: C

Solution:

Given that $|\text{adj } A| = x$ and $|\text{adj } B| = y$, we are tasked with finding $\left|(\text{adj}(AB))^{-1}\right|$.

First, recognize that the determinant of the adjugate of a matrix can be expressed with respect to the determinants of the matrix itself. Using this relation, we have:

$|\text{adj}(AB)| = |\text{adj}(\text{adj}(A))| \cdot |\text{adj}(\text{adj}(B))|$

The inverse of the adjugate adjoint product can be decomposed as:

$\left|(\text{adj}(AB))^{-1}\right| = \left|(\text{adj } B)^{-1}(\text{adj } A)^{-1})\right|$

Because $|\text{adj } A| = x$ and $|\text{adj } B| = y$, the properties of determinants give us:

$\left|(\text{adj } A)^{-1}\right| = \frac{1}{|\text{adj } A|} = \frac{1}{x}$

$\left|(\text{adj } B)^{-1}\right| = \frac{1}{|\text{adj } B|} = \frac{1}{y}$

Thus, the determinant of the inverse of the product is:

$\left|(\text{adj}(AB))^{-1}\right| = \frac{1}{x} \times \frac{1}{y} = \frac{1}{xy}$

Therefore, the result is $\frac{1}{xy}$.

Question16

The system of equations $x+3by+bz=0, x+2ay+az=0$ and $x+4cy+cz=0$ has

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Options:

A. only zero solution for any values of a, b, c

B. non-zero solution for any values of a, b, c

C. non-zero solution, whenever $b(a+c)=2ac$

D. non-zero solution, wherever $a+c=2b$

Answer: C

Solution:

Given, $x+3by+bz=0$

$x+2ay+az=0$

$x+4cy+cz=0$

$\Rightarrow \Delta=\begin{vmatrix}1 & 3b & b \\ 1 & 2a & a \\ 1 & 4c & c\end{vmatrix}$

$\Delta=2ac+3ab-2ab+4bc-4ac-3bc$

$\Delta=-2ac+ab+bc$

For non-zero solution $\Delta=0$

$\Rightarrow 2ac=ab+bc$

Question17

$$\begin{vmatrix}\frac{-bc}{a^2} & \frac{c}{a} & \frac{b}{a} \\ \frac{c}{b} & -\frac{ac}{b^2} & \frac{a}{b} \\ \frac{b}{c} & \frac{a}{c} & -\frac{ab}{c^2}\end{vmatrix}=$$

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Options:

A. 0

B. 4

C. -1

D. $\frac{a^2+b^2+c^2}{a^2b^2c^2}$

Answer: B

Solution:

To evaluate the determinant of the matrix:

$$\begin{vmatrix} \frac{-bc}{a^2} & \frac{c}{a} & \frac{b}{a} \\ \frac{c}{b} & -\frac{ac}{b^2} & \frac{a}{b} \\ \frac{b}{c} & \frac{a}{c} & -\frac{ab}{c^2} \end{vmatrix}$$

we can simplify the process by factoring out $\frac{1}{a^2}$, $\frac{1}{b^2}$, and $\frac{1}{c^2}$ from rows R_1, R_2, and R_3 respectively. This allows us to focus on a simpler determinant:

$$\Rightarrow \quad \frac{1}{a^2b^2c^2}\begin{vmatrix} -bc & ac & ab \\ bc & -ac & ab \\ bc & ac & -ab \end{vmatrix}$$

Next, apply row operations to simplify further: replace R_2 with $R_2 + R_1$ and R_3 with $R_3 + R_1$:

$$\Rightarrow \quad \frac{1}{a^2b^2c^2}\begin{vmatrix} -bc & ac & ab \\ 0 & 0 & 2ab \\ 0 & 2ac & 0 \end{vmatrix}$$

This matrix is now triangular, allowing us to compute the determinant by multiplying the diagonal elements:

$$\Rightarrow \quad \frac{1}{a^2b^2c^2} \times (4a^2b^2c^2) = 4$$

Thus, the value of the determinant is 4.

Question18

If $A = \begin{bmatrix} x & y & y \\ y & x & y \\ y & y & x \end{bmatrix}$ is a matrix such that $5A^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$, then $A^2 - 4A =$

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Options:

A. $5A^{-1}$

B. 51

C. 0

D. 1

Answer: B

Solution:

Given, $5A^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$

and

$$A = \begin{bmatrix} x & y & y \\ y & x & y \\ y & y & x \end{bmatrix}$$

$$\Rightarrow \quad 5A^{-1}A = 5I$$

$$\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} \begin{bmatrix} x & y & y \\ y & x & y \\ y & y & x \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -3x+4y & -y+2x & -y+2x \\ -y+2x & -3x+4y & -y+2x \\ -y+2x & -y+2x & -3x+4y \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\Rightarrow \quad y = 2x \text{ and } -3x+4y = 5$$

$$\Rightarrow \quad 5x = 5$$

$$\Rightarrow \quad x = 1$$

Put $x = 1$, then $y = 2$

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$A^2 - 4A = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4 \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 5I$$

Question19

If $A = \begin{bmatrix} 9 & 3 & 0 \\ 1 & 5 & 8 \\ 7 & 6 & 2 \end{bmatrix}$ **and** $AA^T - A^2 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$**, then** $\sum_{\substack{1 \leq i \leq 3 \\ 1 \leq j \leq 3}} a_{ij} =$

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Options:

A. 35

B. 0

C. 33

D. 1

Answer: A

Solution:

Given, $A = \begin{bmatrix} 9 & 3 & 0 \\ 1 & 5 & 8 \\ 7 & 6 & 2 \end{bmatrix}$

and $AA^T - A^2 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Now,

$$A^2 = \begin{bmatrix} 9 & 3 & 0 \\ 1 & 5 & 8 \\ 7 & 6 & 2 \end{bmatrix} \begin{bmatrix} 9 & 3 & 0 \\ 1 & 5 & 8 \\ 7 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 84 & 42 & 24 \\ 70 & 76 & 56 \\ 83 & 63 & 52 \end{bmatrix}$$

$$AA^T - A^2 = \begin{bmatrix} 9 & 3 & 0 \\ 1 & 5 & 8 \\ 7 & 6 & 2 \end{bmatrix} \begin{bmatrix} 9 & 1 & 7 \\ 3 & 5 & 6 \\ 0 & 8 & 2 \end{bmatrix}$$

$$-\begin{bmatrix} 84 & 42 & 24 \\ 70 & 76 & 56 \\ 83 & 63 & 52 \end{bmatrix}$$

$$= \begin{bmatrix} 90 & 24 & 81 \\ 24 & 90 & 53 \\ 81 & 53 & 89 \end{bmatrix} - \begin{bmatrix} 84 & 42 & 24 \\ 70 & 76 & 56 \\ 83 & 63 & 52 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -18 & 57 \\ -46 & 14 & -3 \\ -2 & -10 & 37 \end{bmatrix}$$

$$\sum_{\substack{1 \leq i \leq 3 \\ 1 \leq j \leq 3}} a_{ij} = 6 - 18 + 57 - 46 + 14$$

Question20

If $a \neq b \neq c, \Delta_1 = \begin{bmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{bmatrix}, \Delta_2 = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{bmatrix}$ **and** $\frac{\Delta_1}{\Delta_2} = \frac{6}{11}$**,**
then $11(a+b+c) =$

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Options:

A. 0

B. 1

C. $ab + bc + ca$

D. $6(ab + bc + ca)$

Answer: D

Solution:

Given,

$$\Delta_1 = \begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Delta_1 = \begin{vmatrix} 1 & a^2 & bc \\ 0 & b^2 - a^2 & c(a-b) \\ 0 & c^2 - a^2 & b(a-c) \end{vmatrix}$$

$$= (a-b)(c-a) \begin{vmatrix} 1 & a^2 & bc \\ 0 & -(a+b) & c \\ 0 & c+a & -b \end{vmatrix}$$

$$= (a-b)(c-a)\left(ab + b^2 - c^2 - ac\right)$$

$$= (a-b)(c-a)(a(b-c) + (b-c)(b+c)$$

$$\Delta_1 = (a-b)(b-c)(c-a)(a+b+c)\ldots$$

$$\text{and } \Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

$C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a^2 & b^2 - a^2 & c^2 - a^2 \\ a^3 & b^3 - a^3 & c^3 - a^3 \end{vmatrix}$$

$$= -(a-b)(c-a)$$

$$\begin{vmatrix} 1 & 0 & 0 \\ a^2 & a+b & c+a \\ a^3 & (a^2+b^2+ab) & c^2+a^2+ac \end{vmatrix}$$

$|a \quad (a + b + ab) \quad c + a + ac|$

$C_2 \to C_2 - C_3$

$= -(a-b)(c-a)$

$$\begin{vmatrix} 1 & 0 & 0 \\ a^2 & b-c & c+a \\ a^3 & (b^2-c^2)+a(b-c) & c^2+a^2+ac \end{vmatrix}$$

$= -(a-b)(b-c)(c-a)$

$$\begin{vmatrix} 1 & 0 & 0 \\ a^2 & 1 & c+a \\ a^3 & b+c+a & c^2+a^2+ac \end{vmatrix}$$
$= -(a-b)(b-c)(c-a)$

$[c^2+a^2+ac-bc-c^2-ac-ab-ac-a^2]$

$= -(a-b)(b-c)(c-a)[-bc-ab-ac]$

$\Delta_2 = (a-b)(b-c)(c-a)[ab+bc+ca]$

From Ist and 2nd equation replace Δ_1 and Δ_2

$\frac{\Delta_1}{\Delta_2} = \frac{6}{11} \Rightarrow \frac{a+b+c}{ab+bc+ca} = \frac{6}{11}$

$\therefore 11(a+b+c) = 6(ab+bc+ca)$

Question21

The system of equations $x + 3y + 7 = 0$, $3x + 10y - 3z + 18 = 0$ and $3y - 9z + 2 = 0$ has

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Options:

A. unique solution.

B. infinitely many solutions.

C. no solution.

D. finite number of solution.

Answer: C

Solution:

Given, $x + 3y + 7 = 0$

$3x + 10y - 3z + 18 = 0$

$3y - 9z + 2 = 0$

$$D = \begin{vmatrix} 1 & 3 & 0 \\ 3 & 10 & -3 \\ 0 & 3 & -9 \end{vmatrix} = 0$$

$$D_x = \begin{vmatrix} -7 & 3 & 0 \\ -18 & 10 & -3 \\ -2 & 3 & -9 \end{vmatrix} = 99$$

$$D_y = \begin{vmatrix} 1 & -7 & 0 \\ 3 & -18 & -3 \\ 0 & -2 & -9 \end{vmatrix} = -33$$

$$D_z = \begin{vmatrix} 1 & 3 & -7 \\ 3 & 10 & -18 \\ 0 & 3 & -2 \end{vmatrix} = -11$$

As $D = 0$, and none D_x, D_y and D_z is zero, the given system of equation has no solution.

Question22

If α, β and γ are the roots of the equation $\begin{vmatrix} x & 2 & 2 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix} = 0$ and $\min(\alpha, \beta, \gamma) = \alpha$, then $2\alpha + 3\beta + 4\gamma$ is equal to

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Options:

A. 6

B. 8

C. -6

D. -8

Answer: A

Solution:

We have, α, β, γ are the roots of the equation $\begin{vmatrix} x & 2 & 2 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix} = 0$ and

$\min(\alpha, \beta, \gamma) = \alpha$

Now, $\begin{vmatrix} x & 2 & 2 \\ 2 & x & 2 \\ 2 & 2 & x \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} x-2 & 0 & 2 \\ 0 & x-2 & 2 \\ 2-x & 2-x & x \end{vmatrix} = 0$

$[C_1 \rightarrow C_1 - C_3 \text{ and } C_2 \rightarrow C_2 - C_3]$

$$\Rightarrow \quad (x-2)^2 \begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ -1 & -1 & x \end{vmatrix} = 0$$

$$\left[C_1 \rightarrow \frac{1}{x-2} C_1 \text{ and } C_2 \rightarrow \frac{1}{x-2} C_2\right]$$

$$\Rightarrow (x-2)^2[1(x+2) - 0 + (-1)(0-2)] = 0$$

$$\Rightarrow (x-2)^2(x+4) = 0 \Rightarrow x = -4, 2, 2$$

$$\therefore \quad \alpha = -4, \beta = 2 \text{ and } \gamma = 2$$

$$\text{Now, } 2\alpha + 3\beta + 4\gamma$$

$$= 2(-4) + 3(2) + 4(2)$$

$$= -8 + 6 + 8 = 6$$

Question23

If A $= \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ and A$^{-1} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then $\sum_{\substack{1 \leq i \leq 3 \\ 1 \leq j \leq 3}} a_{ij} =$

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Options:

A.

$\frac{2}{3}$

B. $\frac{1}{3}$

C. 1

D.

Answer: B

Solution:

We have,

$$A=\begin{bmatrix}1 & 2 & 2\\ 3 & 2 & 3\\ 1 & 1 & 2\end{bmatrix} \text{ and } A^{-1}=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$$

$$|A|=1(4-3)-2(6-3)+2(3-2)=1-6+2=-3$$

$$C_{11}=\begin{vmatrix}2 & 3\\ 1 & 2\end{vmatrix}=1$$

$$C_{12}=-\begin{vmatrix}3 & 3\\ 1 & 2\end{vmatrix}=-3$$

$$C_{13}=\begin{vmatrix}3 & 2\\ 1 & 1\end{vmatrix}=1$$

$$C_{21}=-\begin{vmatrix}2 & 2\\ 1 & 2\end{vmatrix}=-2$$

$$C_{22}=\begin{vmatrix}1 & 2\\ 1 & 2\end{vmatrix}=0$$

$$C_{23}=-\begin{vmatrix}1 & 2\\ 1 & 1\end{vmatrix}=1$$

$C_{31} = \begin{vmatrix} 2 & 2 \\ 2 & 3 \end{vmatrix} = 2, C_{32} = -\begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} = 3$

$C_{33} = \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = -4$

$$\text{adj } A = \begin{bmatrix} 1 & -3 & 1 \\ -2 & 0 & 1 \\ 2 & 3 & -4 \end{bmatrix}^T = \begin{bmatrix} 1 & -2 & 2 \\ -3 & 0 & 3 \\ 1 & 1 & -4 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}(\text{adj } A) = \frac{1}{-3}\begin{bmatrix} 1 & -2 & 2 \\ -3 & 0 & 3 \\ 1 & 1 & -4 \end{bmatrix}$$

Now,

$\sum_{\substack{1 \leq i \leq 3 \\ 1 \leq j \leq 3}} a_{ij} = \left(-\frac{1}{3}\right)$

$[1 + (-2) + 2 + (-3) + 0 + 3 + 1 + 1 + (-4)]$

$= \left(-\frac{1}{3}\right)(-1) = \frac{1}{3}$

Question24

If $AX = D$ represents the system of linear equations $3x - 4y + 7z + 6 = 0, 5x + 2y - 4z + 9 = 0$ and $8x - 6y - z + 5 = 0$, then

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Options:

A. $\text{Rank}(A) = \text{Rank}([AD]) = 1$

B. $\text{Rank}(A) = \text{Rank}([AD]) = 2$

C. $\text{Rank}(A) = \text{Rank}([AD]) = 3$

D. $\text{Rank } (A) \neq \text{Rank}([AD])$

Answer: C

Solution:

We have,

$AX = D$, where

$$A = \begin{bmatrix} 3 & -4 & 7 \\ 5 & 2 & -4 \\ 8 & -6 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } D = \begin{bmatrix} -6 \\ -9 \\ -5 \end{bmatrix}$$

$$|A| = 3(-2-24) + 4(-5+32) + 7(-30-16)$$

$$= -78 + 108 - 322$$

$$= -292 \neq 0$$

$\therefore$ Rank (A) = Order of $A = 3$

$$AD = \begin{bmatrix} 3 & -4 & 7 \\ 5 & 2 & -4 \\ 8 & -6 & -1 \end{bmatrix} \begin{bmatrix} -6 \\ -9 \\ -5 \end{bmatrix} = \begin{bmatrix} -17 \\ -28 \\ 11 \end{bmatrix}$$

Clearly, Rank $([AD]) = 3$

Hence, Rank $(A) = \text{Rank}([AD]) = 3$

Question25

If $(x, y, z) = (\alpha, \beta, \gamma)$ is the unique solution of the system of simultaneous linear equations $3x - 4y + z + 7 = 0$, $2x + 3y - z = 10$ and $x - 2y - 3z = 3$, then $\alpha =$

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Options:

A. 3

B. -3

C. -1

D. 1

Answer: D

Solution:

We have,

$(x, y, z) = (\alpha, \beta, \gamma)$ is a unique solution of the system of simultaneous linear equations

$3x - 4y + z = -7$

$2x + 3y - z = 10$

$x - 2y - 3z = 3$

so, by cramer's rule,

$$x = \alpha = \frac{\begin{vmatrix} -7 & -4 & 1 \\ 10 & 3 & -1 \\ 3 & -2 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & -4 & 1 \\ 2 & 3 & -1 \\ 1 & -2 & -3 \end{vmatrix}}$$

$$= \frac{-7(-9-2) + 4(-30+3) + (-20-9)}{3(-9-2) + 4(-6+1) + (-4-3)}$$

$$= \frac{77 - 108 - 29}{-33 - 20 - 7}$$

$$= \frac{-60}{-60} = 1$$

Question26

If α, β, γ are the roots of the equation $2x^3 - 5x^2 + 4x - 3 = 0$, then $\Sigma\alpha\beta(\alpha + \beta) =$

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Options:

A. 8

B. 4

C. 2

D. $\frac{1}{2}$

Answer: D

Solution:

Given,

α, β, γ are the roots of the equation

$$2x^3 - 5x^2 + 4x - 3 = 0$$

$$\therefore \alpha + \beta + \gamma = \frac{-(-5)}{2} = \frac{5}{2} \quad \ldots \text{(i)}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{4}{2} = 2 \quad \ldots \text{(ii)}$$

$$\alpha\beta\gamma = \frac{-(-3)}{2} = \frac{3}{2} \quad \ldots \text{(iii)}$$

Now, $\Sigma\alpha\beta(\alpha + \beta)$

$$= \alpha\beta(\alpha + \beta) + \beta\gamma(\beta + \gamma) + \gamma\alpha(\alpha + \gamma)$$

$$= \alpha^2\beta + \alpha\beta^2 + \beta^2\gamma + \beta\gamma^2 + \alpha^2\gamma + \alpha\gamma^2$$

$$= (\alpha + \beta + \gamma)(\alpha\beta + \beta\gamma + \gamma\alpha) - 3\alpha\beta\gamma$$

$$= \left(\frac{5}{2}\right)(2) - 3\left(\frac{3}{2}\right)$$

[using Eqs. (i), (ii) and (iii)]

$= \frac{10}{2} - \frac{9}{2} = \frac{1}{2}$

Question27

A, B, C and D are square matrices such that $A + B$ is symmetric, $A - B$ is skew-symmetric and D is the transpose of C.If

$A=\begin{bmatrix}-1 & 2 & 3\\ 4 & 3 & -2\\ 3 & -4 & 5\end{bmatrix}$ **and** $C=\begin{bmatrix}0 & 1 & -2\\ 2 & -1 & 0\\ 0 & 2 & 1\end{bmatrix}$**, then the matrix** $B+D=$

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Options:

A. $\begin{bmatrix}-1 & 6 & 3\\ 6 & 2 & -2\\ 3 & -2 & 6\end{bmatrix}$

B. $\begin{bmatrix}-1 & 6 & 3\\ 3 & 2 & -2\\ 1 & -2 & 6\end{bmatrix}$

C. $\begin{bmatrix}3 & 2 & -2\\ 2 & 6 & 3\\ -2 & 3 & 2\end{bmatrix}$

D. $\begin{bmatrix}1 & -2 & 6\\ -2 & 3 & 2\\ 6 & 2 & 1\end{bmatrix}$

Answer: B

Solution:

$\because A+B$ is symmecric

$\Rightarrow \quad (A+B)^T = A+B$

$\Rightarrow \quad A^T + B^T = A+B \quad \ldots$ (i)

and $A - B$ is skew-symmetric

$\Rightarrow \quad (A - A^T = -(A-B)$

$A^T - B^T = -A + B \quad \ldots$ (ii)

From Eqs. (i) and (ii), we get

$B = A^T$

$$\Rightarrow B = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 3 & -2 \\ 3 & -4 & 5 \end{bmatrix}^r = \begin{bmatrix} -1 & 4 & 3 \\ 2 & 3 & -4 \\ 3 & -2 & 5 \end{bmatrix}$$

Given D is the transpose of C

$$\Rightarrow \quad = C^r = \begin{bmatrix} 0 & 1 & -2 \\ 2 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}^T$$

$$D = \begin{bmatrix} 0 & 2 & 0 \\ 1 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

$$50, B + D = \begin{bmatrix} -1 & 4 & 3 \\ 2 & 3 & -4 \\ 3 & -2 & 5 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 0 \\ 1 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 6 & 3 \\ 3 & 2 & -2 \\ 1 & -2 & 6 \end{bmatrix}$$

Question28

If A is square matrix and $A^2 + I = 2A$, then $A^9 =$

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Options:

A. $8A^2 - 71$

B. $9A + 81$

C. $9A - 8I$

D. $8A^2 + 7I$

Answer: C

Solution:

We have, $A^2 + I = 2A \Rightarrow A^2 = 2A - I$

$\because A^9 = A^2 \cdot A^2 A^2 A^2 A$

$= (2A - I)(2A - I)(2A - I)(2A - I)A$

$\Rightarrow A^9 = \left(4A^2 + I - 4A\right)\left(4A^2 + I - 4A\right)A$

$= (4A - 3I)(4A - 3I)A$

$= \left(16A^2 + 9I - 24A\right)A$

$= (8A - 7I)A$

$= 8A^2 - 7A$

$= 16A - 8I - 7A$

$= 9A - 8I$

Question29

$$\det \begin{bmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{bmatrix} =$$

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Options:

A. $(a-b)(b-c)(c-a)$

B. $(a+b)(b+c)(c+a)$

C. $2abc$

D. $4abc$

Answer: D

Solution:

$$\text{Let } \Delta = \begin{bmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{bmatrix}$$

On multiply R_1, R_2 and R_3 by c, a and b, respectively

$$\Rightarrow \Delta = \frac{1}{abc} \begin{bmatrix} a^2+b^2 & c^2 & c^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{bmatrix}$$

Apply $R_1 \to R_1 - (R_2 + R_3)$

$$\therefore \Delta=\frac{1}{abc}\begin{bmatrix}0 & -2b^2 & -2a^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2\end{bmatrix}$$

$$=\frac{-2}{abc}\begin{bmatrix}0 & b^2 & a^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2\end{bmatrix}$$

Apply $R_2 \rightarrow R_2-R_1$ and $R_3 \rightarrow R_3-R_1$, we get

$$=\frac{-2}{abc}\begin{bmatrix}0 & b^2 & a^2 \\ a^2 & c^2 & 0 \\ b^2 & 0 & c^2\end{bmatrix}$$

$$=\frac{-2}{abc}\left[0-b^2\left(a^2c^2-0\right)+a^2\left(0-b^2c^2\right)\right]$$

$$=\frac{-2}{abc}\left(-2a^2b^2c^2\right)=4abc$$

Question30

The system of simultaneous linear equations

$$x-2y+3z=4, 3x+y-2z=7$$
$$2x+3y+z=6 \text{ has}$$

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Options:

A. infinitely many solutions.

B. no solution.

C. unique solution having $z=2$.

D. unique solution having $z=\frac{1}{2}$.

Answer: D

Solution:

We have,

$x-2y+3z=4$

$3x+y-2z=7$

$2x+3y+z=6$

Here,

$$D=\begin{vmatrix}1 & -2 & 3\\ 3 & 1 & -2\\ 2 & 3 & 1\end{vmatrix}$$

$$=1(1+6)+2(3+4)+3(9-2)=42$$

$$D_x=\begin{vmatrix}4 & -2 & 3\\ 7 & 1 & -2\\ 6 & 3 & 1\end{vmatrix}$$

$$=4(1+6)+2(7+12)+3(21-6)=111$$

$$D_y=\begin{vmatrix}1 & 4 & 3\\ 3 & 7 & -2\\ 2 & 6 & 1\end{vmatrix}=3$$

$$D_z=\begin{vmatrix}1 & -2 & 4\\ 3 & 1 & 7\\ 2 & 3 & 6\end{vmatrix}=21$$

So, system of linear equations has unique solution and $z=\frac{D_z}{D}=\frac{21}{42}=\frac{1}{2}$

Question31

$$\begin{vmatrix} \sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} & 5 & \sqrt{10} \\ 3 & \sqrt{15} & 5 \end{vmatrix} =$$

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Options:

A. $5\sqrt{2} - 3\sqrt{3}$

B. $5\sqrt{3} - 3\sqrt{5}$

C. $10\sqrt{3} - 15\sqrt{2}$

D. $15\sqrt{2} - 25\sqrt{3}$

Answer: D

Solution:

$$\begin{vmatrix} \sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} & 5 & \sqrt{10} \\ 3 & \sqrt{15} & 5 \end{vmatrix}$$

Taking $\sqrt{3}, \sqrt{5}$ and $\sqrt{5}$ from C_1, C_2 and C_3, we get

$$\sqrt{3} \times \sqrt{5} \times \sqrt{5} \begin{vmatrix} 1 & 2 & 1 \\ \sqrt{5} & \sqrt{5} & \sqrt{2} \\ \sqrt{3} & \sqrt{3} & \sqrt{5} \end{vmatrix}$$

$$= 5\sqrt{3} \begin{vmatrix} 1 & 1 & 1 \\ \sqrt{5} & 0 & \sqrt{2} \\ \sqrt{3} & 0 & \sqrt{5} \end{vmatrix}$$

(Applying $C_2 \rightarrow C_2 - C_1$,

$-5\sqrt{3}(5 - \sqrt{6})$

$= -25\sqrt{3} + 15\sqrt{2} = 15\sqrt{2} - 25\sqrt{3}$

Question32

If A is a non-singular matrix such that $(A-2I)(A-3I)=0$, then $\frac{1}{5}A+\frac{6}{5}A^{-1}=$

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Options:

A. 0

B. I

C. 2I

D. 3I

Answer: B

Solution:

Given that A is a non-singular matrix, and it satisfies the equation:

$(A-2I)(A-3I)=0$

Since A is non-singular, it implies that $|A| \neq 0$, which means A^{-1} exists.

Expanding the given equation:

$A^2-2A-3A+6I=0$

Simplifies to:

$A^2-5A+6I=0$

Multiplying both sides of this equation by A^{-1} yields:

$A-5I+6A^{-1}=0$

Dividing through by 5 gives:

$\frac{A}{5}+\frac{6}{5}A^{-1}=I$

Question33

Let A be a matrix such that AB is a scalar matrix, where $B=\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ and $\det(3A)=27$. Then, $3A^{-1}+A^2=$

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Options:

A. $\begin{bmatrix} 4 & -6 \\ 0 & 2 \end{bmatrix}$

B. $\begin{bmatrix} 9 & -4 \\ 0 & 3 \end{bmatrix}$

C. $\begin{bmatrix} 10 & -6 \\ 0 & 2 \end{bmatrix}$

D. $\begin{bmatrix} 10 & -6 \\ 0 & 4 \end{bmatrix}$

Answer: D

Solution:

Given that $AB =$ Scalar matrix

$= \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

where $\lambda =$ scalar

Also, given that $B = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ and

$\det(3A) = 27$

$\Rightarrow \quad \det A = \frac{27}{9} = 3$

Now, let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\therefore \quad AB = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & 2a+3b \\ c & 2c+3d \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

Comparing on both sides, we get

$$a = \lambda, c = 0, 2a + 3b = 0, 2c + 3d = \lambda$$

$$\therefore \quad b = -\frac{2\lambda}{3} \text{ and } d = \frac{\lambda}{3}$$

Thus, $A = \begin{bmatrix} \lambda & -\frac{2\lambda}{3} \\ 0 & \frac{\lambda^3}{3} \end{bmatrix}$

$$|A| = \begin{vmatrix} \lambda & -2\lambda \\ 0 & \frac{\lambda}{3} \end{vmatrix} = \frac{\lambda^2}{3} = 3 \quad \text{(given)}$$

$$\Rightarrow \quad \lambda^2 = 9 \Rightarrow \lambda = \pm 3$$

$$\therefore \quad A^2 = \begin{bmatrix} \lambda & -\frac{2\lambda}{3} \\ 0 & \frac{\lambda^2}{3} \end{bmatrix} \begin{bmatrix} \lambda & -\frac{2\lambda}{3} \\ 0 & \frac{\lambda^2}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \lambda^2 & -\frac{2\lambda^2}{3} - \frac{2\lambda^2}{9} \\ 0 & \frac{\lambda^2}{9} \end{bmatrix} = \begin{bmatrix} \lambda^2 & -\frac{8\lambda^2}{9} \\ 0 & \frac{\lambda^2}{9} \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 9 & -8 \\ 0 & 1 \end{bmatrix}$$

Clearly, $A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

$$\Rightarrow \quad 3A^{-1} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

Thus,

$$3A^{-1} + A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} 9 & -8 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 10 & -6 \\ 0 & 4 \end{bmatrix}$$

Question34

If A is a symmetric matrix with real entries, then

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Options:

A. A^{-1} is symmetric, if it exists

B. A^{-1} always exists and is symmetric

C. A^{-1} is skew-symmetric, if it exists

D. A^{-1} always exists and is skew-symmetric

Answer: A

Solution:

If A is a symmetric matrix with real entries, it means $A^T = A$.

Now, suppose the inverse A^{-1} exists.

For symmetric matrices, the transpose of the inverse is equal to the inverse of the transpose, so:

$(A^{-1})^T = (A^T)^{-1} = A^{-1}$

This equation shows that A^{-1} is also symmetric. Therefore, if the inverse of a symmetric matrix exists, it is necessarily symmetric as well.

Question35

If $\omega \neq 1$ is a cube root of unity, then

$$\begin{vmatrix} \omega+\omega^2 & \omega^2+\omega^9 & \omega^9+\omega \\ \omega^{27}+\omega^{31} & \omega^{31}+\omega^{17} & \omega^{17}+\omega^{27} \\ \omega^{30}+\omega^{41} & \omega^{41}+\omega^{19} & \omega^{19}+\omega^{30} \end{vmatrix} =$$

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Options:

A. 3

B. 2

C. 1

D. 0

Answer: D

Solution:

$$\begin{vmatrix} \omega+\omega^2 & \omega^2+\omega^9 & \omega^9+\omega \\ \omega^{27}+\omega^{31} & \omega^{31}+\omega^{17} & \omega^{17}+\omega^{27} \\ \omega^{30}+\omega^{41} & \omega^{41}+\omega^{19} & \omega^{19}+\omega^{30} \end{vmatrix}$$

$$=\begin{vmatrix} \omega & \omega^2 & \omega^9 \\ \omega^{27} & \omega^{31} & \omega^{17} \\ \omega^{30} & \omega^{41} & \omega^{19} \end{vmatrix}+\begin{vmatrix} \omega^2 & \omega^9 & \omega \\ \omega^{31} & \omega^{17} & \omega^{27} \\ \omega^{41} & \omega^{19} & \omega^{30} \end{vmatrix}$$

$$=\omega \times \omega^{17} \times \omega^{19}\begin{vmatrix} 1 & \omega & \omega^8 \\ \omega^{10} & \omega^{14} & 1 \\ \omega^{11} & \omega^{22} & 1 \end{vmatrix}+\omega \times \omega^{17} \times \omega^{19}\begin{vmatrix} \omega & \omega^8 & 1 \\ \omega^{14} & 1 & \omega^{10} \\ \omega^{22} & 1 & \omega^{11} \end{vmatrix}$$

$$=\omega^{37}\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & 1 \end{vmatrix}+\omega^{37}\begin{vmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ \omega & 1 & \omega^2 \end{vmatrix}$$

$$=\omega^{37} \times 0+\omega^{37} \times 0$$

$$=0+0=0$$

Question36

If P is a non-singular matrix such that $I+P+P^2+\ldots\ldots+P^n=0(0$ denotes the null matrix $)$, then $P^{-1}=$

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Options:

A. P^n

B. $-P^n$

C. $-(1+P+\ldots\ldots+P^n)$

D. $-\left(1+P+\ldots\ldots+P^{n-1}\right)$

Answer: A

Solution:

Given a non-singular matrix P such that:

$I+P+P^2+\ldots+P^n=0$

we aim to determine P^{-1}.

First, recognize the expression as a series whose sum is zero:

$I+P+P^2+\ldots+P^n=0$

This expression can be seen as a finite geometric series. Now, apply the formula for the sum of a geometric series:

$S = I\frac{P^{n+1}-I}{P-I}$

Given that the entire series equals zero:

$\frac{P^{n+1}-I}{P-I} = 0$

This implies:

$P^{n+1} - I = 0$

Thus, we have:

$P^{n+1} = I$

This equation indicates that P raised to the power $n+1$ results in the identity matrix I. Therefore, multiplying both sides by P^{-1}, we deduce:

$P^{n+1} \cdot P^{-1} = I \cdot P^{-1}$

Hence:

$P^n = P^{-1}$

Therefore, the inverse of P is P^n.

Question37

If $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$ **and** $\det\left(A^2\right) = 25$**, then** $|\alpha| =$

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Options:

A. 5

B. 5^2

C. 1

D. $\frac{1}{5}$

Answer: D

Solution:

To solve for $|\alpha|$, we begin with the matrix A:

$$A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$$

Given that $\det(A^2) = 25$, we can derive that $(\det(A))^2 = 25$. Hence, $|A| = \pm 5$.

Next, we compute the determinant of A:

The matrix A is upper triangular. Thus, the determinant $|A|$ is the product of its diagonal elements:

$|A| = 5 \times \alpha \times 5 = 25\alpha$

Given that $|A| = \pm 5$, we equate and solve for α:

$25\alpha = \pm 5$

This implies that $\alpha = \pm \frac{1}{5}$.

Therefore, the absolute value of α is:

$|\alpha| = \frac{1}{5}$

Question38

P is a 3×3 square matrix and $\operatorname{Tr}(P) \neq 0$. If $\operatorname{Tr}\left(P - P^I\right) + \operatorname{Tr}\left(P + P^T\right) + \frac{\operatorname{Tr}(P)}{\operatorname{Tr}(P^T)} + \operatorname{Tr}(P) \times \operatorname{Tr}\left(P^T\right) = 0$, then $\operatorname{Tr}(P) =$

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Options:

A. 0

B. -1

C. 4

D. 3

Answer: B

Solution:

$$P=\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix} \operatorname{Tr}(P)=a+e+i$$

$$P^T=\begin{bmatrix}a & d & g\\ b & e & h\\ c & f & i\end{bmatrix} \operatorname{Tr}\left(P^T\right)=a+e+i$$

$$\Rightarrow \operatorname{Tr}(P)=\operatorname{Tr}\left(P^T\right)$$

$$\operatorname{Tr}\left(P-P^T\right)+\operatorname{Tr}\left(P+P^\tau\right)+\frac{\operatorname{Tr}(P)}{\operatorname{Tr}\left(P^T\right)}+\operatorname{Tr}(P)$$

$$\times \operatorname{Tr}\left(P^T\right)=0$$

$$\Rightarrow \operatorname{Tr}(P)-\operatorname{Tr}\left(P^T\right)+\operatorname{Tr}(P)+\operatorname{Tr}\left(P^T\right)+\frac{\operatorname{Tr}(P)}{\operatorname{Tr}(P)} + \operatorname{Tr}(P)\cdot\operatorname{Tr}(P)=0$$

$$\Rightarrow (\operatorname{Tr}(P))^2+2\operatorname{Tr}(P)+1=0$$

$$(\operatorname{Tr}(P)+1)^2=0$$

$$\operatorname{Tr}(P)=-1$$

Question39

If the system of equations

$x+ky+3z=-2$,

$4x+3y+kz=14$,

$2x+y+2z=3$ **can be solved by matrix inversion method, then**

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Options:

A. $k \neq 0$ and $\frac{9}{2}$

B. $k=0$ or $\frac{9}{2}$

C. $k \neq \frac{1}{2}$ and 2

D. $k=\frac{1}{2}$ or 2

Answer: A

Solution:

To solve the given system of equations using the matrix inversion method, we need to ensure that the coefficient matrix is invertible. This requires that the determinant of the matrix is not equal to zero.

Consider the system of equations:

$x + ky + 3z = -2$

$4x + 3y + kz = 14$

$2x + y + 2z = 3$

The coefficient matrix for this system is:

$$\begin{bmatrix} 1 & k & 3 \\ 4 & 3 & k \\ 2 & 1 & 2 \end{bmatrix}$$

To check the invertibility, we calculate the determinant of this matrix:

Determinant $= 1(6 - k) - 4(2k - 3) + 2(k^2 - 9)$

Simplifying the expression:

$1(6 - k) = 6 - k$

$-4(2k - 3) = -8k + 12$

$2(k^2 - 9) = 2k^2 - 18$

Combining these results gives:

$6 - k - 8k + 12 + 2k^2 - 18 = 2k^2 - 9k$

For the matrix to be invertible, we need the determinant to be non-zero:

$2k^2 - 9k \neq 0$

This expression can be factored as:

$k(2k - 9) \neq 0$

Thus, the values of k for which the system is solvable by matrix inversion are:

$k \neq 0$

$k \neq \frac{9}{2}$

Therefore, k should not be 0 and not $\frac{9}{2}$.
